ASSIGNMENT 2 - MATH 112

(QUESTIONS BASED YOUR CHAPTER 1 OF YOUR TEXTBOOK: SECTIONS 2 AND 3)

(1) Solve each equation using the techniques we've seen in class. (Please be sure to solve some problems using "completing the square" and some using the quadratic equation so that you are familiar with both techniques.)

   (a) $x^2 - 5x + 1 = 0$
   (b) $x^2 = \frac{3}{4}x - \frac{1}{8}$
   (c) $5x^2 - 7x + 5 = 0$
   (d) $6 - 4x^{-1/2} + 2x^{3/2} = 0$
   (e) $\frac{1}{r} + \frac{2}{1-r} = \frac{4}{r^2}$
   (f) $b^2x^2 - 5bx + 4 = 0$ (solve for x and assume that $b \neq 0$)

(2) Without solving the equations, determine the number of real solutions each equation has.

   (a) $x^2 = 6x - 9$
   (b) $x^2 + rx - s = 0$

(3) Find all $k \in \mathbb{R}$ such that the equation $kx^2 + 36x + k = 0$ has exactly one real solution.

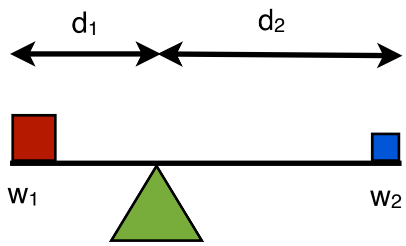
(4) Find 4 consecutive odd integers, whose sum is 416.

(5) Suppose you work at a medical research institute which uses a bleach solution to sterilize its tools. The institute has 100 gallons of solution stored in a 100 gallon barrel and the solution has a concentration of 2% bleach mixed with pure distilled water. New safety guidelines state that in order to completely sterilize tools, the concentration of bleach in the solution should be 5%. How much of the solution should be drained and replaced with bleach in order for the 100 gallons of solution to comply with the new guidelines?

(6) A man 2 meters tall is walking away from a lamp post which is 6 meters high. How long is his shadow when he is 10 meters from the lamp post?

(7) Ana and Brian are spending the summer painting houses. It takes them 18 hours to paint one house if they work together. When Brian works alone, he takes 20% less time to paint one house than the time Ana takes to paint one house when working by herself. How many hours does it take each of them to paint one house when they work individually?

(8) Archimedes (287 BC - 212 BC) was a Greek intellectual who made contributions to many branches of science, including mathematics. In this problem, we will see an equation which he proved, and which led him to remark, "Give me a place to stand on, and I will move the Earth."



$$w_1 d_1 = w_2 d_2$$

(a) Suppose we are given two objects with masses w_1 and w_2, respectively. When placed on a lever, these masses will balance exactly when their distances from the fulcrum of the lever obey the so-called "law of the lever": $w_1 d_1 = w_2 d_2$. Here d_1 is the distance from the fulcrum to the mass w_1 and d_2 is the distance from the fulcrum to the mass w_2, as shown in the above diagram. Solve this equation for w_1, and explain in a few sentences what this equation says about the relationship between the masses and their distances from the fulcrum.

(b) The mass of the earth is approximately 6×10^{24} kg. Suppose you placed the earth on the edge of a plank, 1 meter from a rock positioned under the plank. You would like to stand on the other end of the plank, in order to lift the earth off the ground as Archimedes suggests. Assume that you weigh 60 kg. How far away from the rock would you have to stand in order to balance with the earth?

(c) The diameter of the milky way galaxy is approximately 9×10^{17} km. Would you be able to conduct this balancing act within the confines of our galaxy?

(9) For this problem use the formula $h = -16t^2 + v_0 t$ where h is height measured in feet, t is time measured in seconds, and v_0 is initial velocity.

(a) A ball is thrown straight upward from ground level at an initial speed of $v_0 = 40$ ft/sec.

(i) When does the ball reach a height of 24 ft?

(ii) When does it reach a height of 48 ft?

(iii) What is the greatest height reached by the ball?

(iv) When does the ball reach the highest point of its path?

(v) When does the ball hit the ground?

(b) How fast would a ball have to be thrown upward to reach a maximum height of 100 ft?

(10) The fish population in a lake rises and falls according to the formula

$$F = 1000(30 + 17t - t^2)$$

where F is the number of fish at time t, and t is measured in years since January 1, 2002.

(a) On what date will the fish population again be the same as it was on January 1, 2002?

(b) By what date will all the fish in the lake have died?

(11) *A Faulty Proof:* Written below is a proof of the statement 2=1. See if you can spot the mistake in this "proof".

$$\begin{aligned}
\text{Let } a &= b. \\
\text{Multiply both sides by } a\text{: } a^2 &= ab \\
\text{Add } a^2 \text{ to both sides: } a^2 + a^2 &= a^2 + ab \\
\text{Combine terms on the left: } 2a^2 &= a^2 + ab \\
\text{Subtract } 2ab \text{ from both sides: } 2a^2 - 2ab &= a^2 + ab - 2ab \\
\text{Combine terms on the right: } 2a^2 - 2ab &= a^2 - ab. \\
\text{Factor 2 out from the left side: } 2(a^2 - ab) &= 1(a^2 - ab)
\end{aligned}$$

Divide both sides by $(a^2 - ab)$ to conclude: $2 = 1$.